Parameterized Complexity

or "what I do for a living"

Michał Pilipczuk

Institute of Informatics, University of Warsaw, Poland

Cor Baayen award ceremony,
Budapest, October 26$^{\text{th}}$, 2016

Thanks!

Tatjana Abramovskaya
Anna Adamaszek
Florian Barbero
Ivan Bliznets
Hans L. Bodlaender
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Outline of the talk

- Short introduction to Parameterized Complexity.
- Some concepts, some problems, and some solutions.
  - Graph modification problems.
  - Treewidth, tree decompositions, and their algorithmic usage.
  - Network problems on planar graphs.
- Commercial break!

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- Establishing formal framework $\rightsquigarrow$ Mathematical work on foundations

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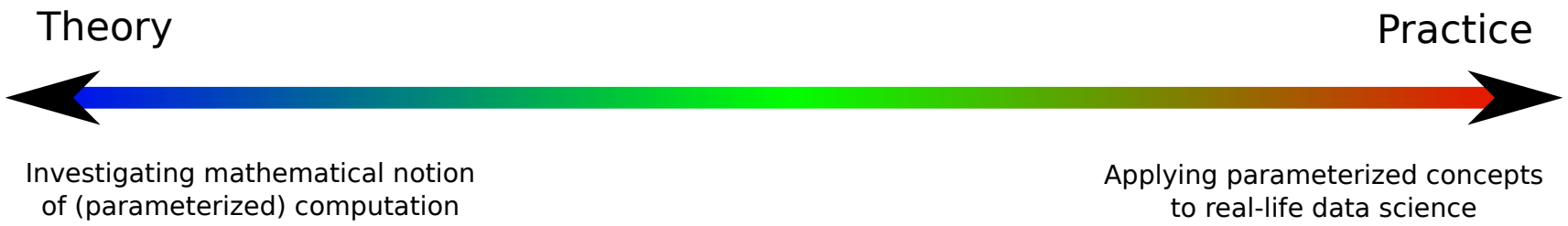
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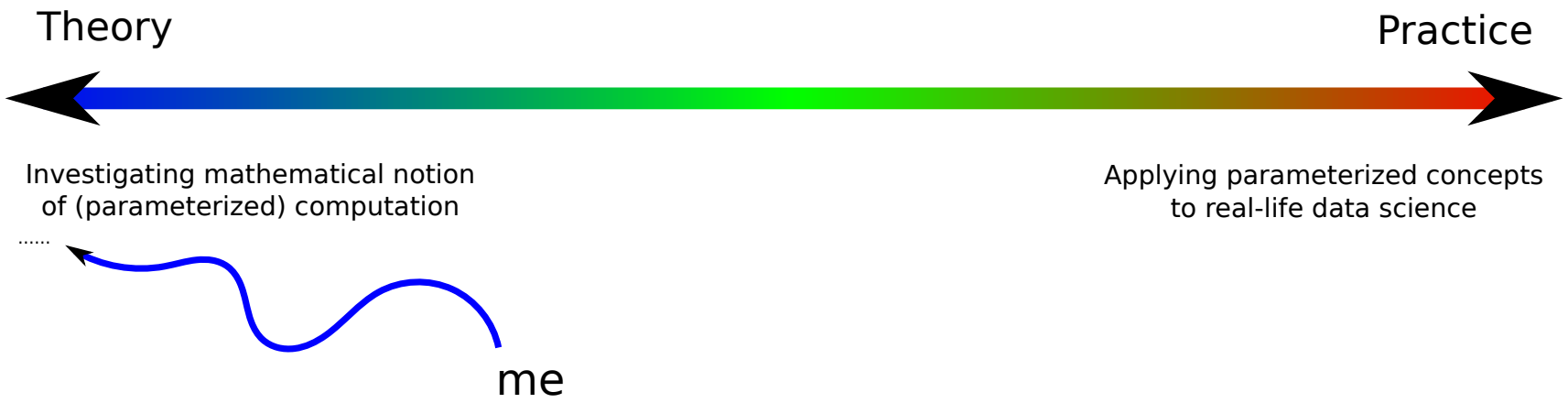
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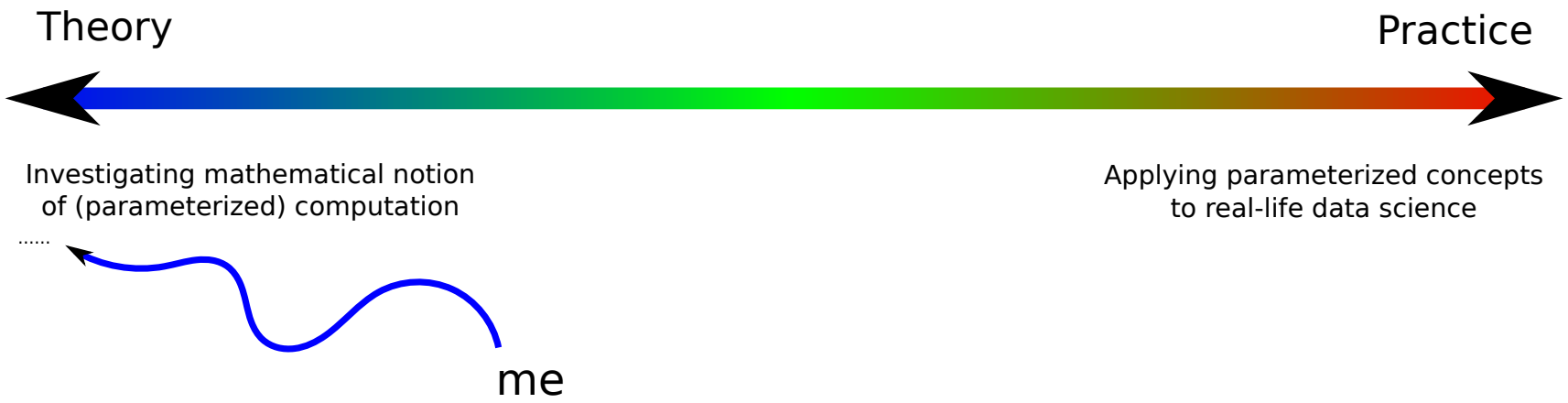

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- **Variants**: Target class of structured graphs, allowed edit operations...

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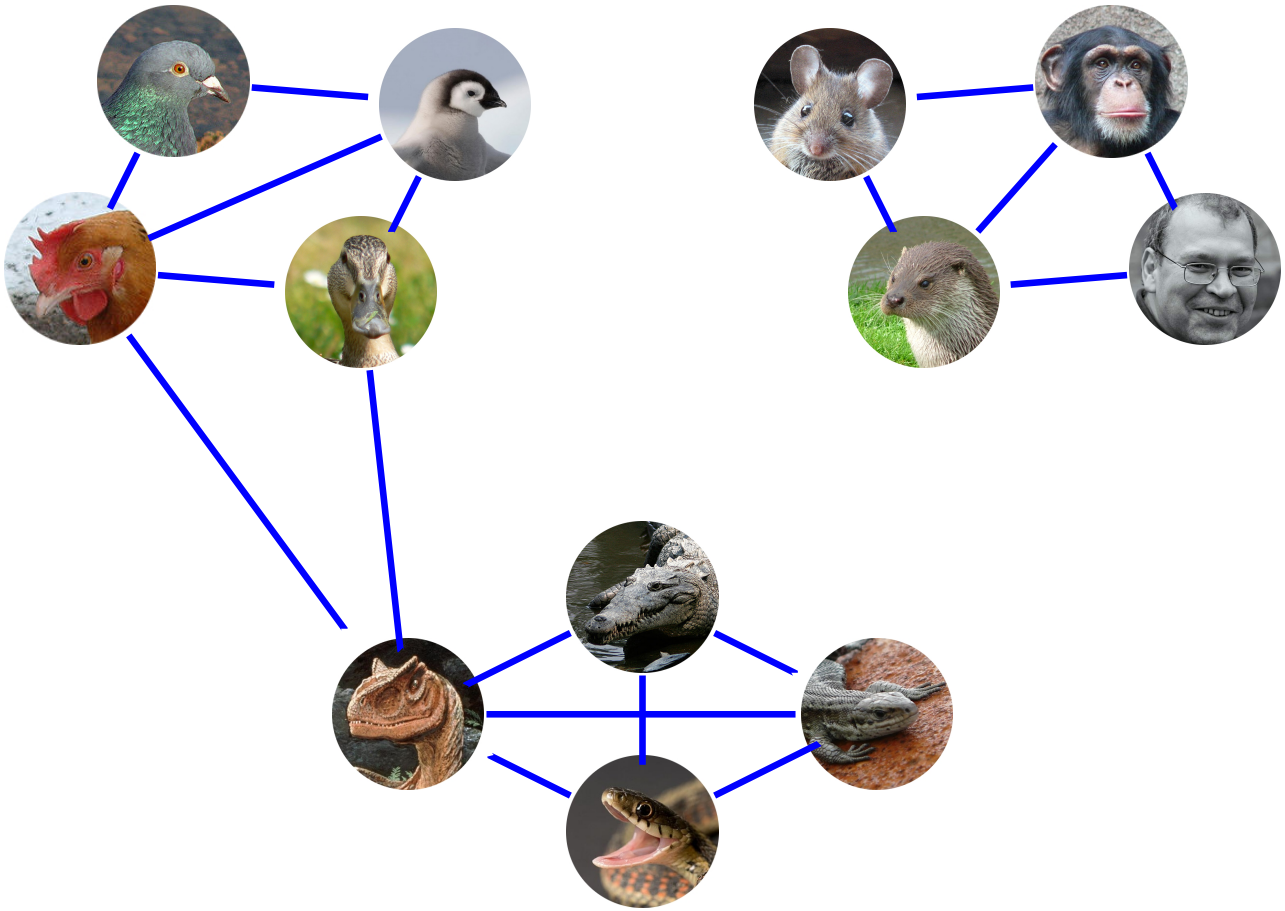

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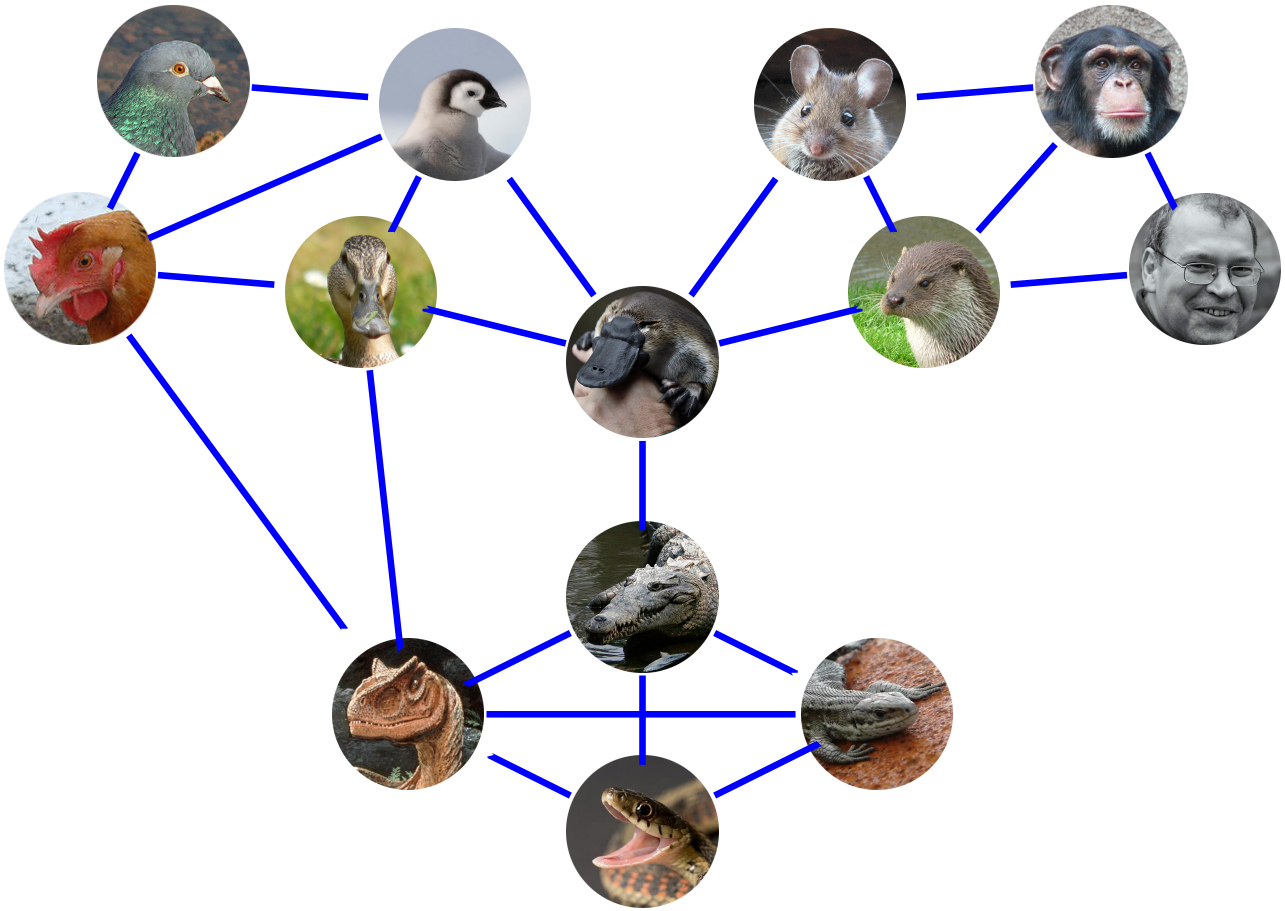

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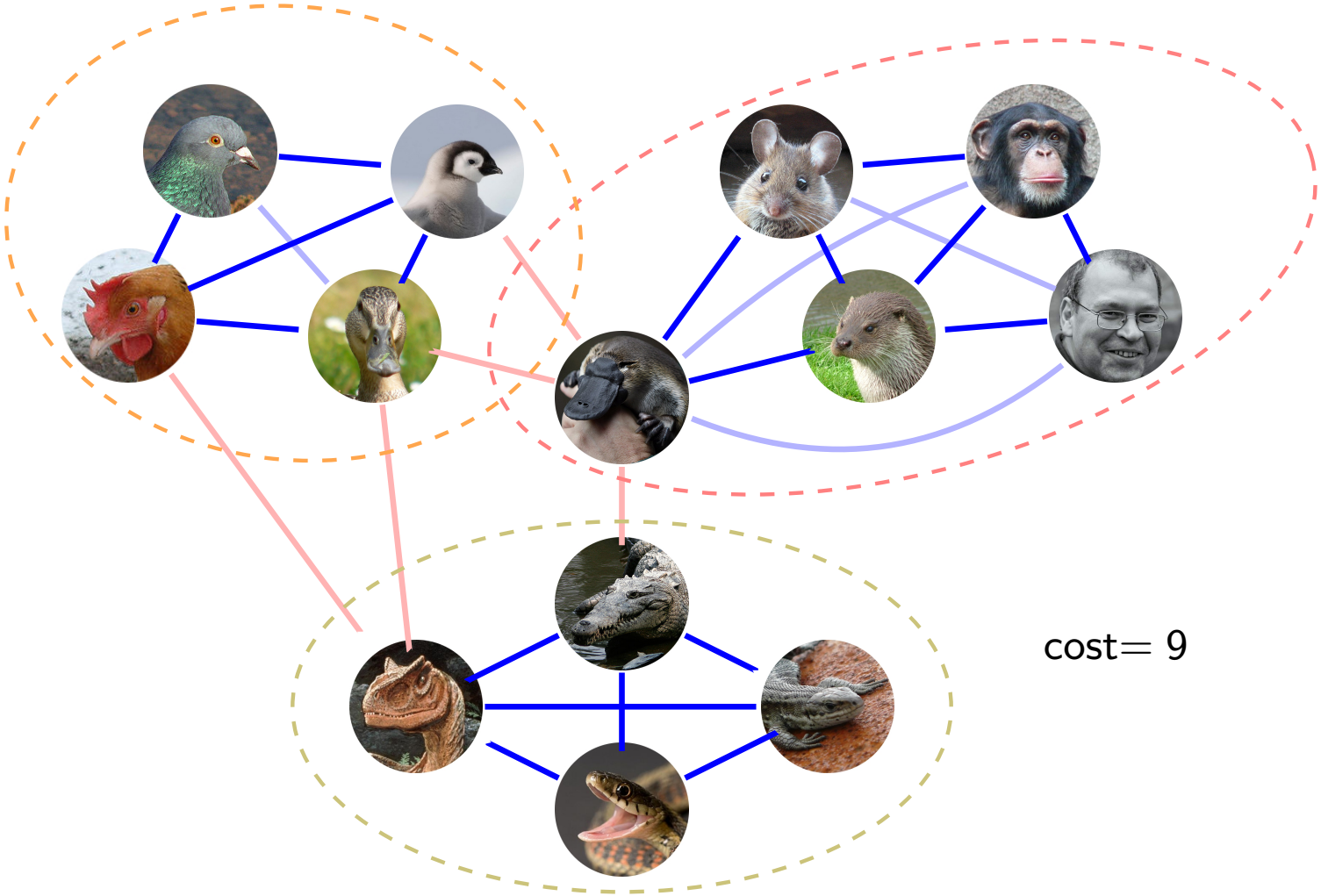

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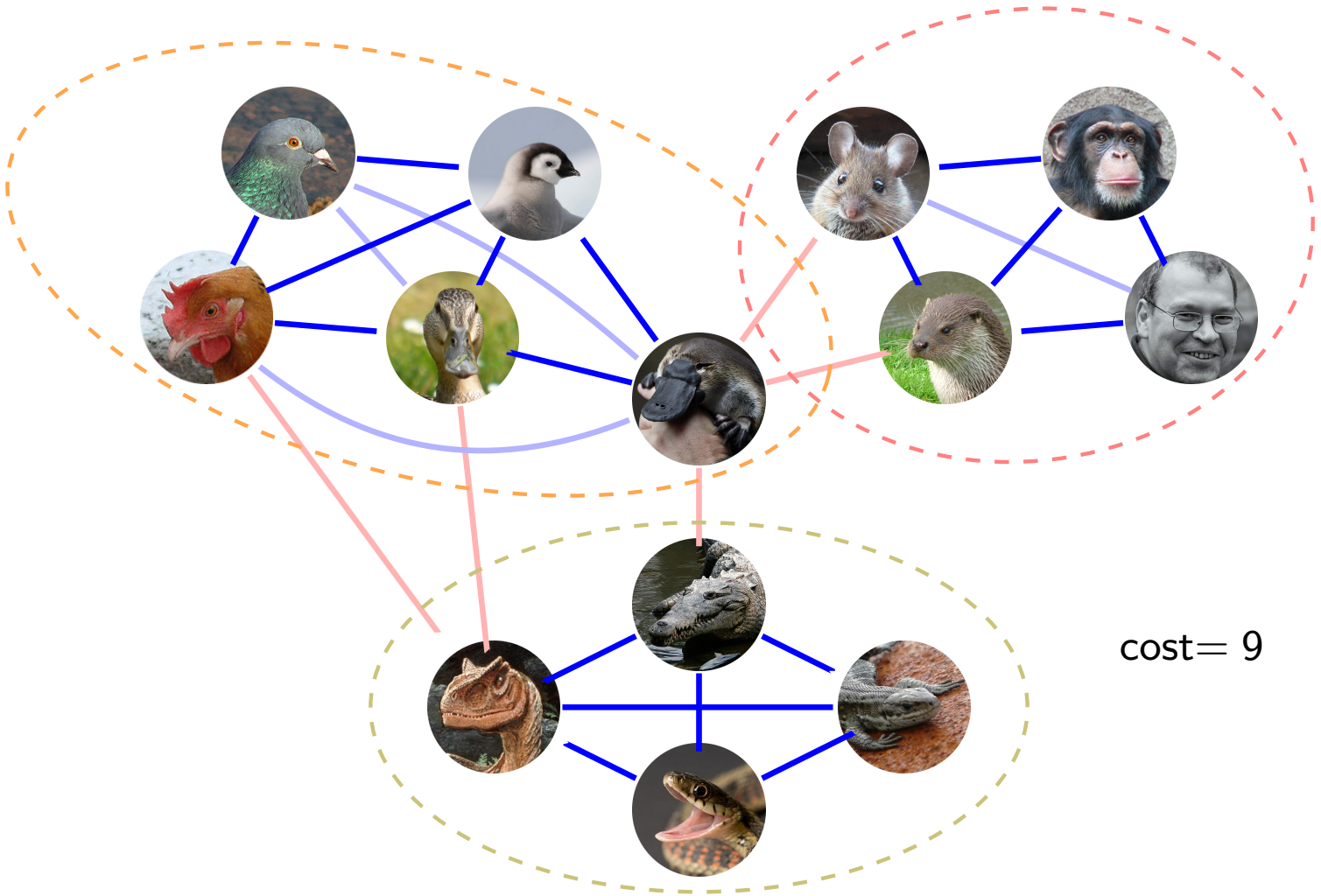

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- What if the target number of clusters p is small compared to k?

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  - An algorithm with running time $2^{o(k)} \cdot n^c$ is unlikely [Komusiewicz, Uhlmann].
- What if the target number of clusters p is small compared to k?

Theorem Fomin, Kratsch, Pilipczuk, Pilipczuk, Villanger; 2013

Cluster Editing can be solved in time $2^{\mathcal{O}(\sqrt{pk})} + \mathcal{O}(n+m)$, where p is the target number of clusters.

Moreover, this running time is likely to be tight for every p as a function of k.

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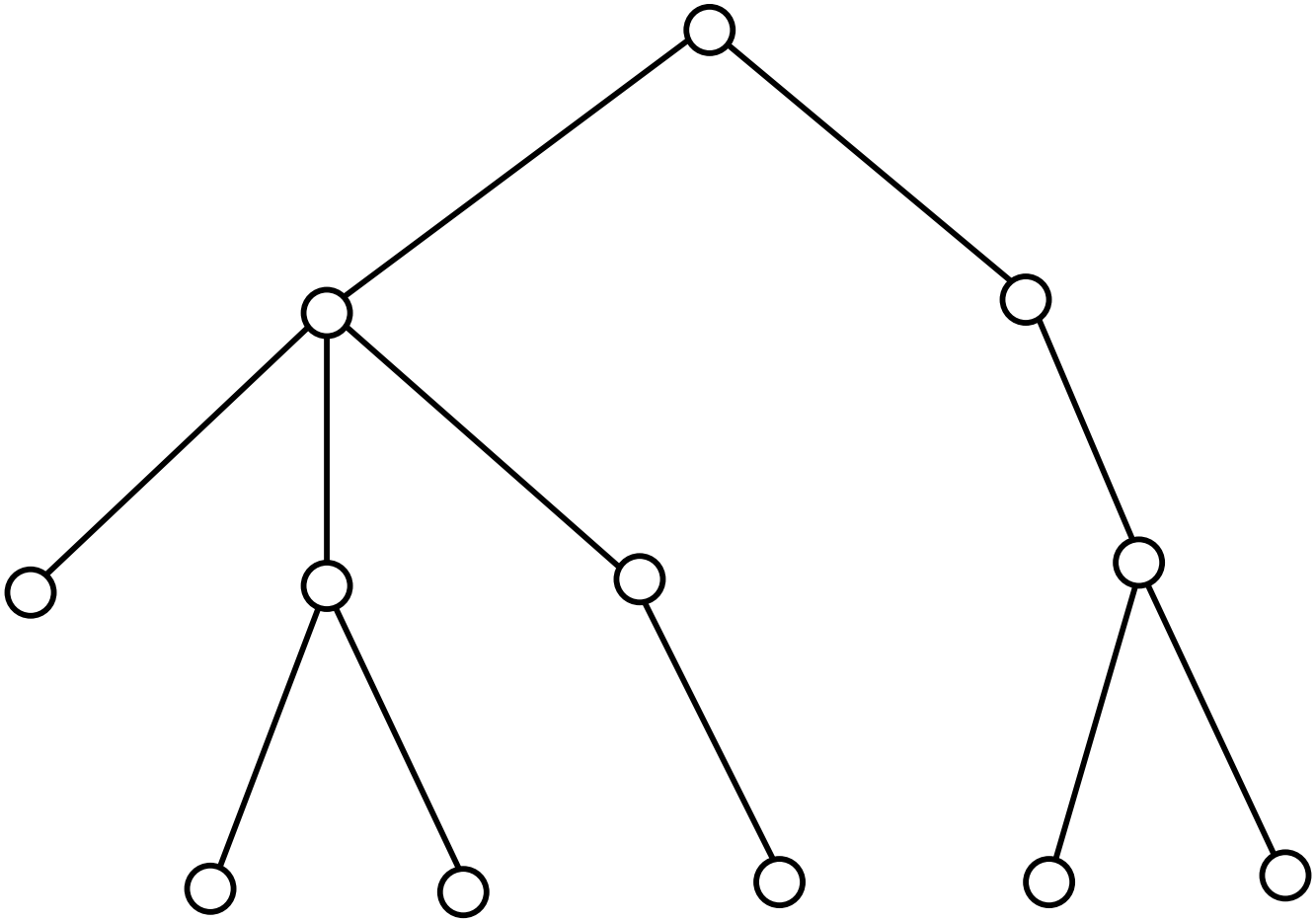
- Standard measure for tree-likeness is the **treewidth** of a graph.

Treelike graphs



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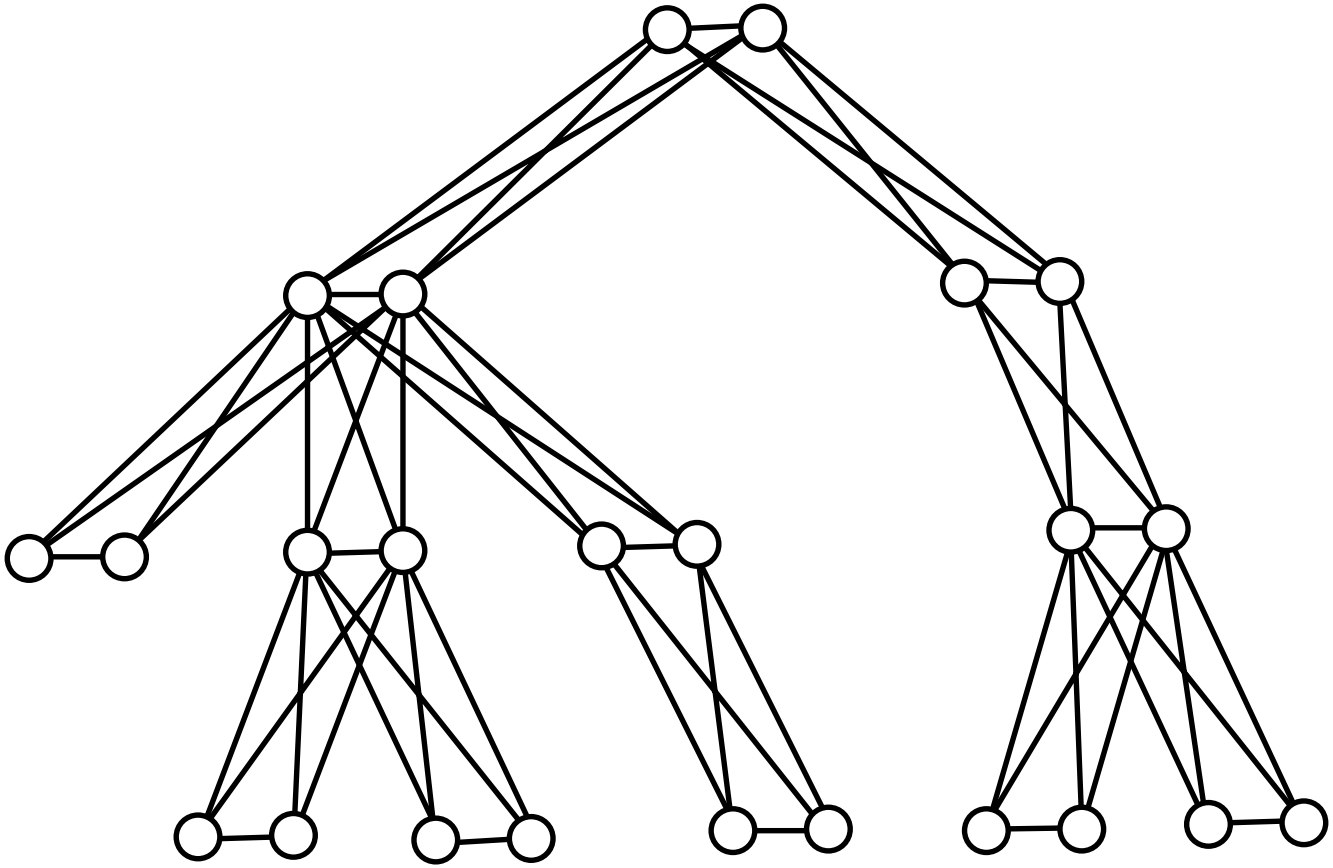

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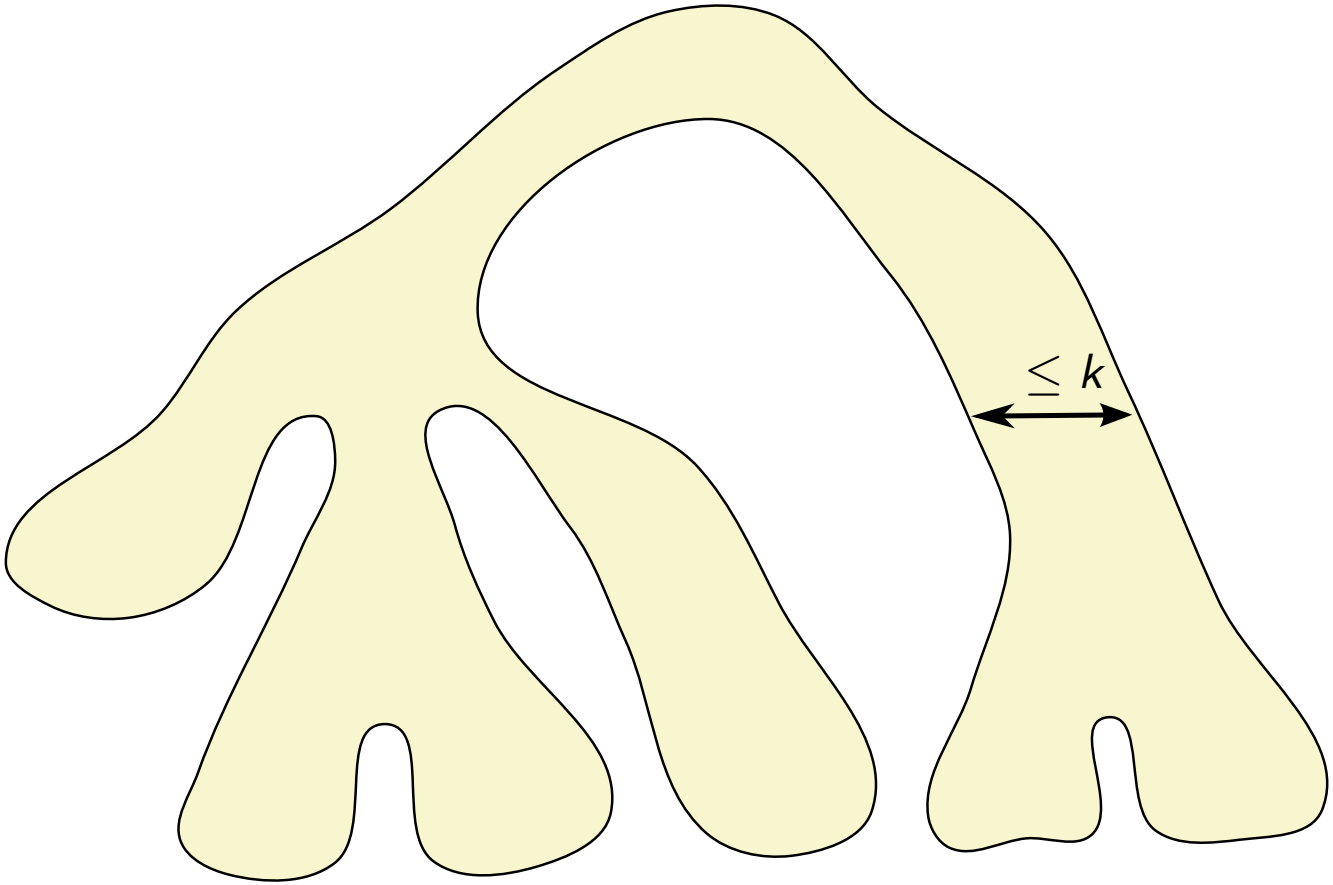

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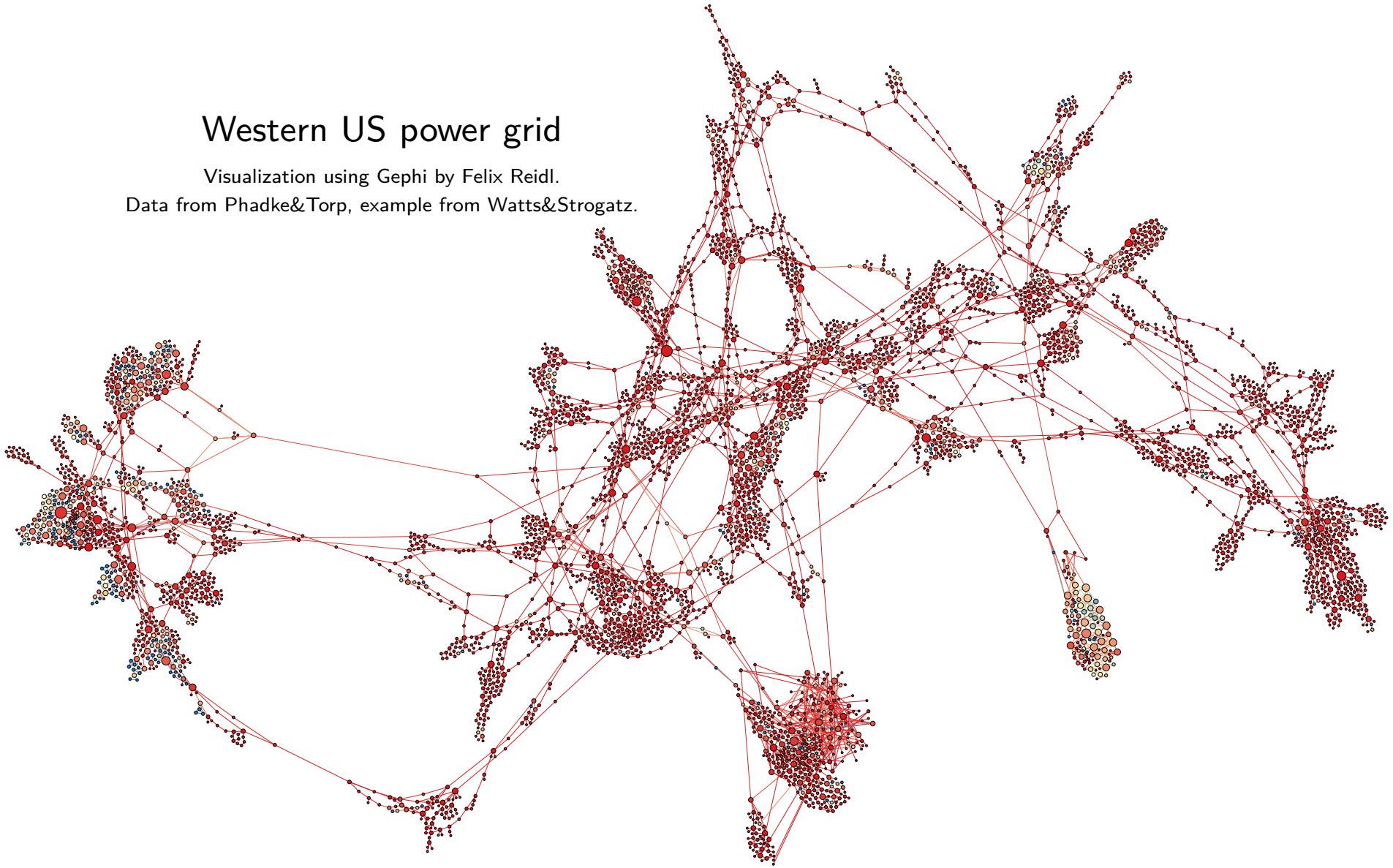

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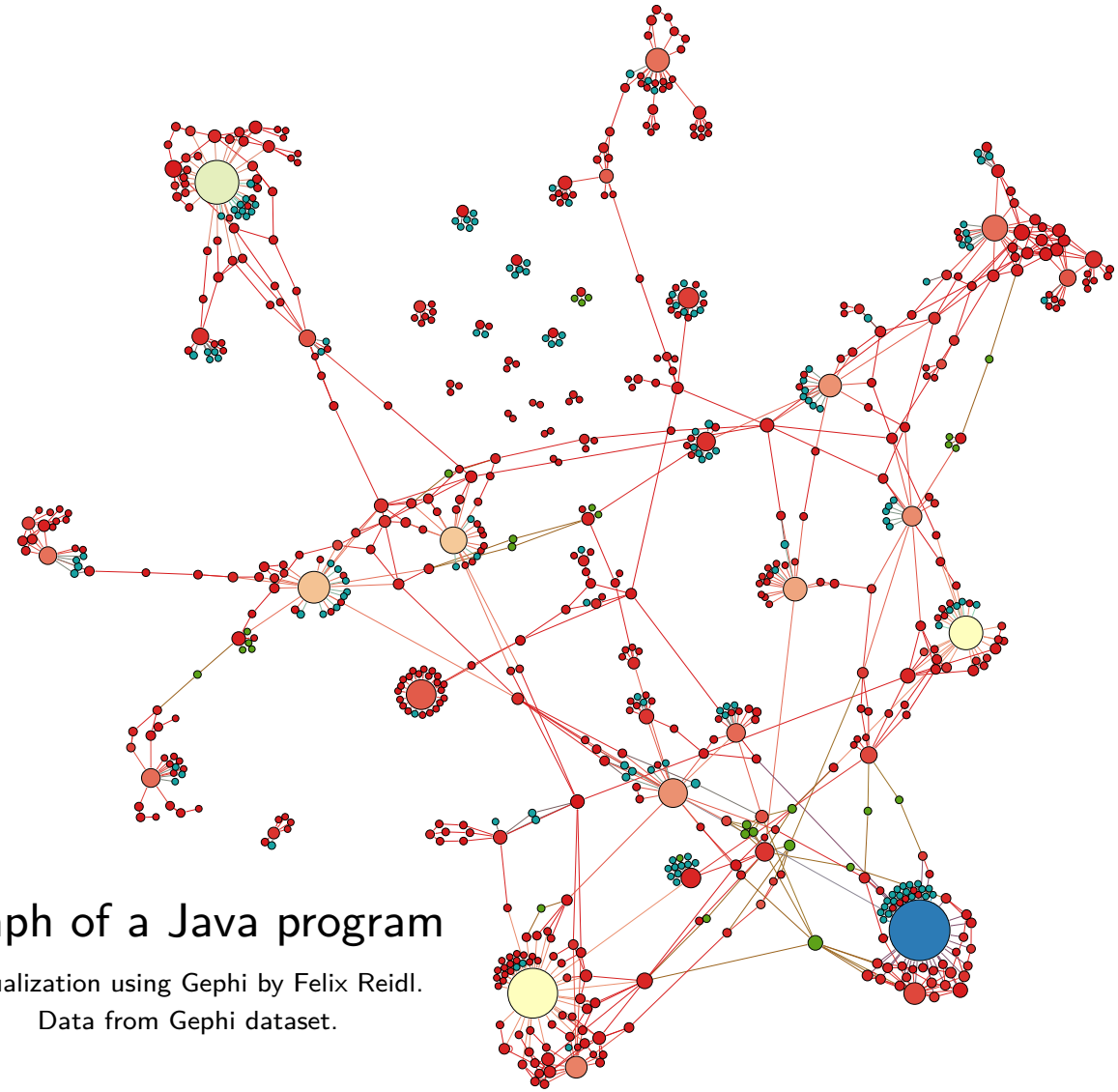

Graph of a Java program

Visualization using Gephi by Felix Reidl.
Data from Gephi dataset.

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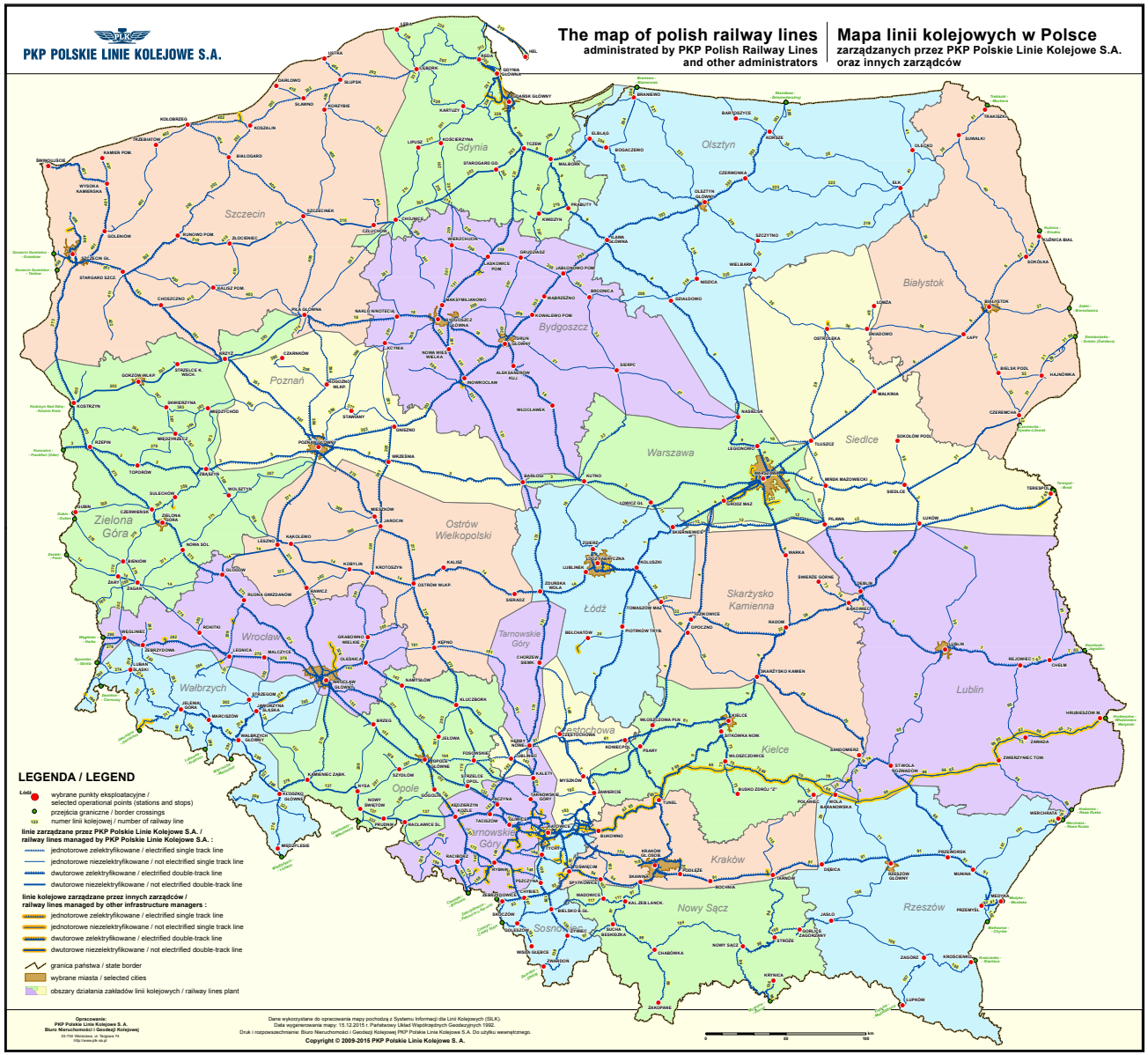
- **Practice**: Need for a reliable library for treewidth-based algorithms.

Steiner Tree



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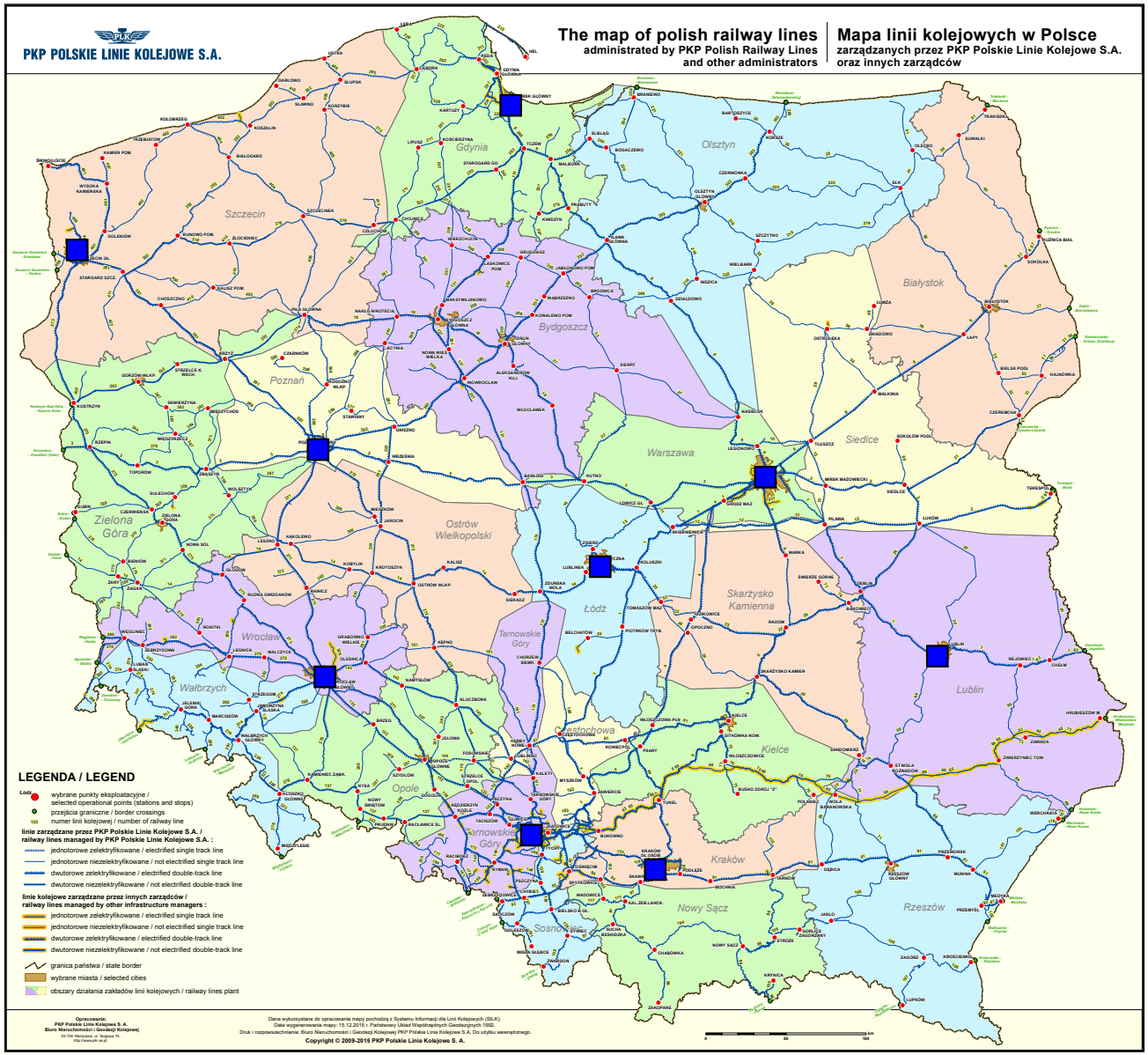

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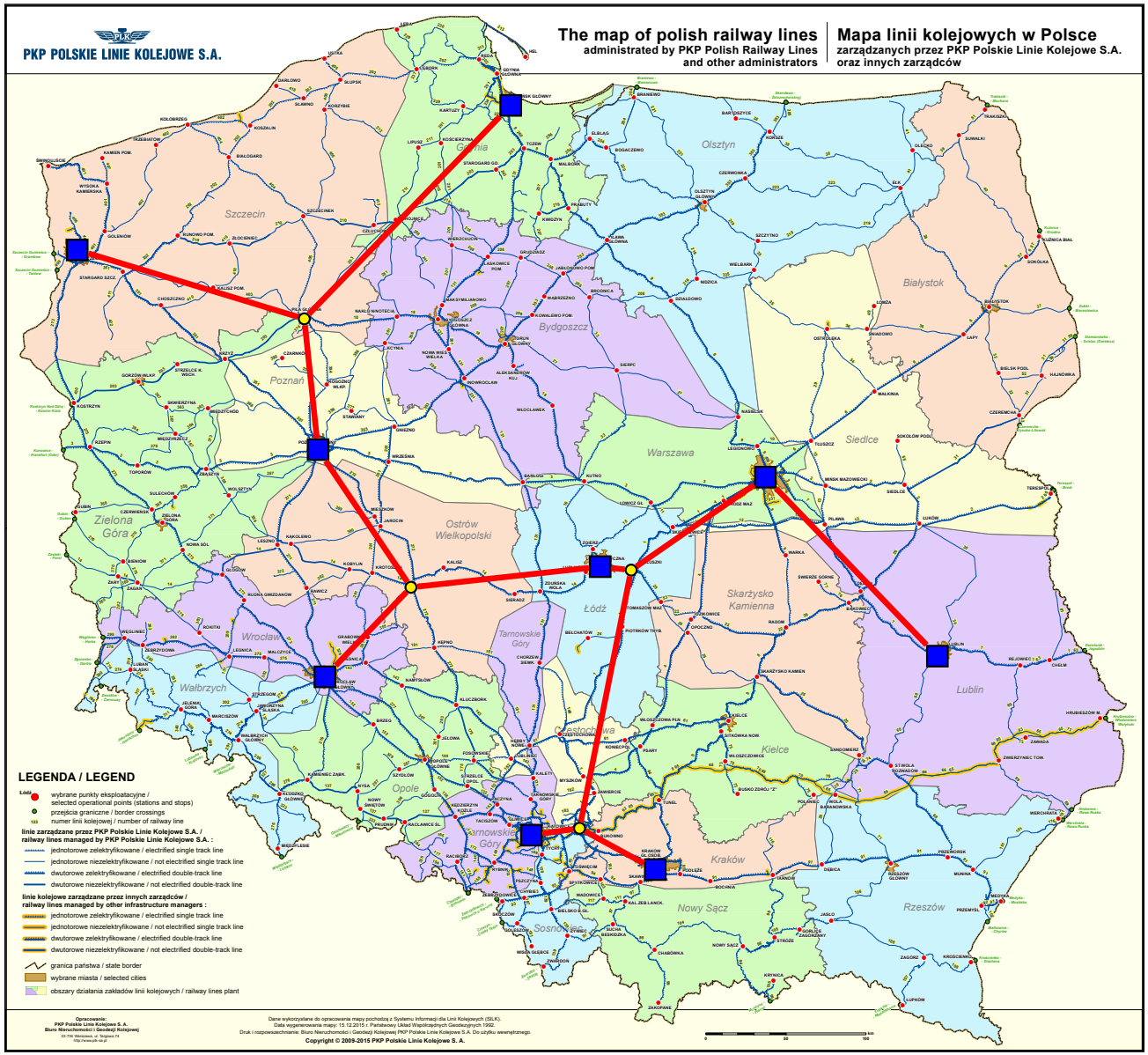

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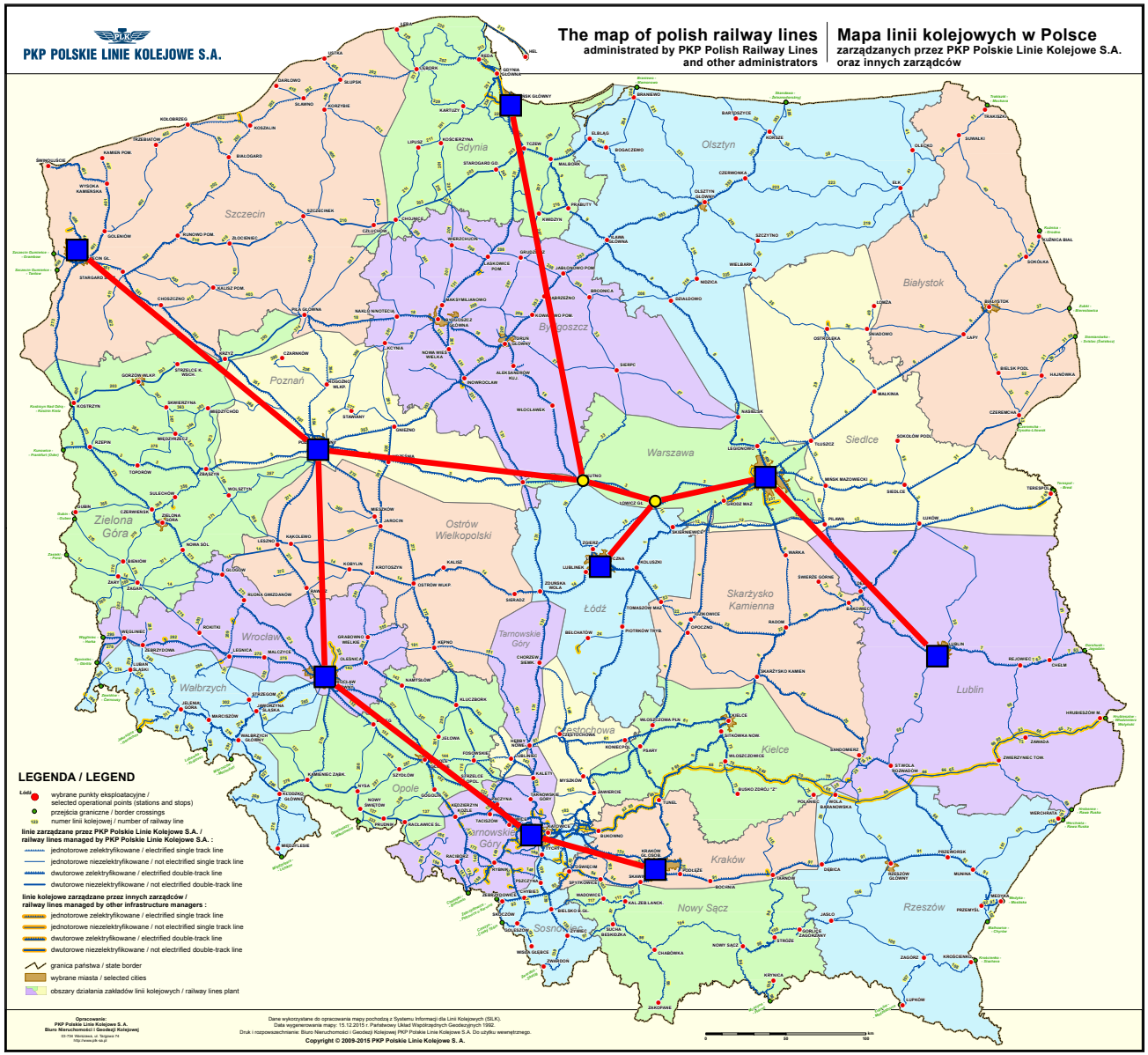

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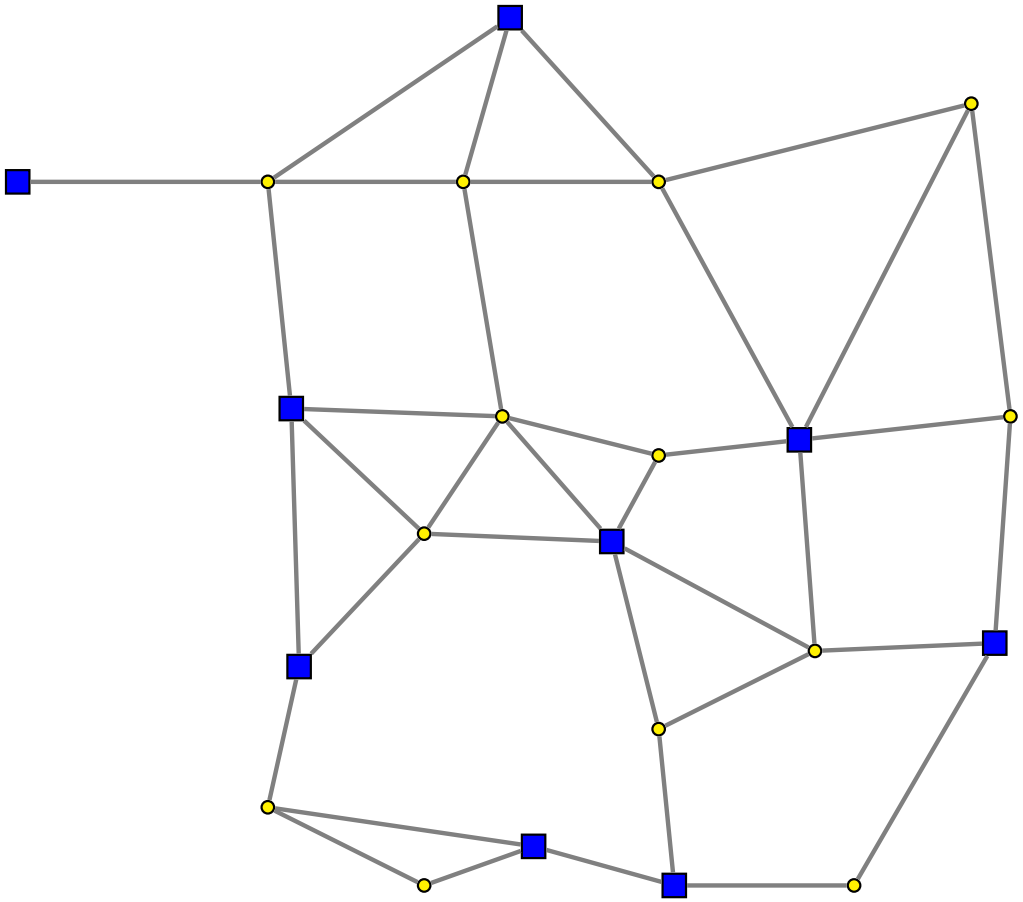

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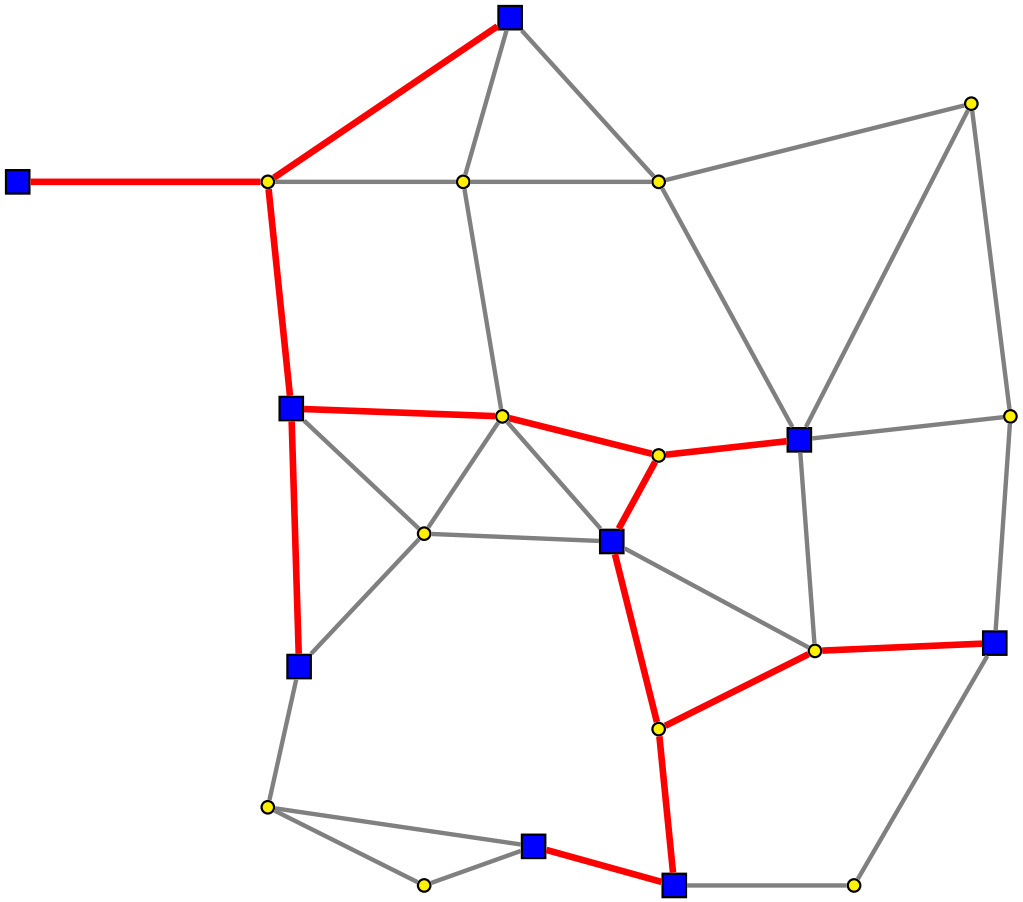

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- Something about the parameterization by $|T|$ coming soon...

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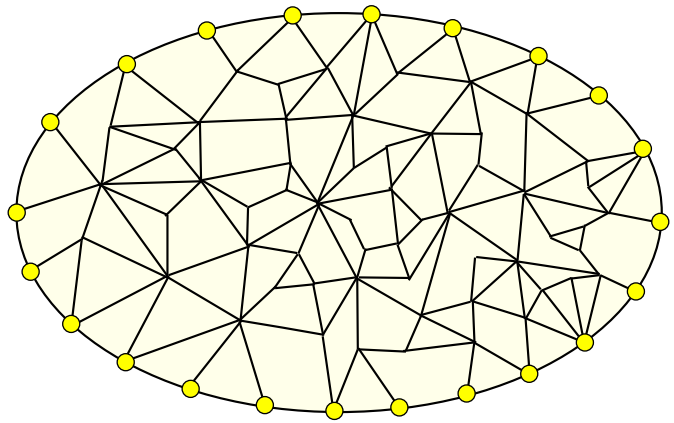

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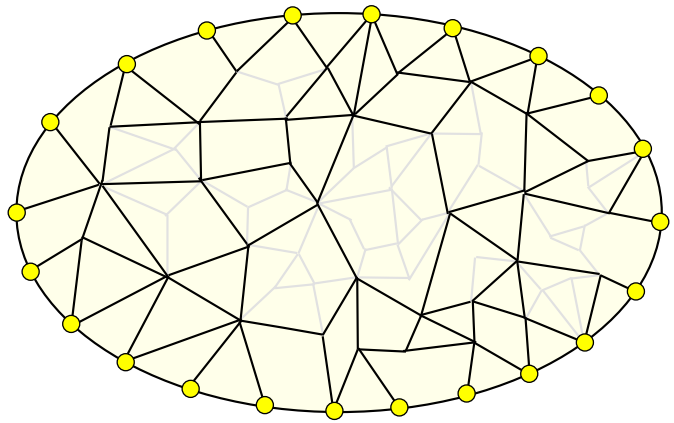

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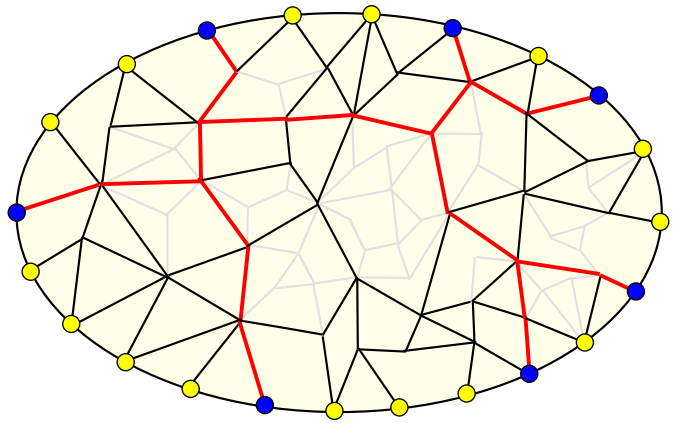

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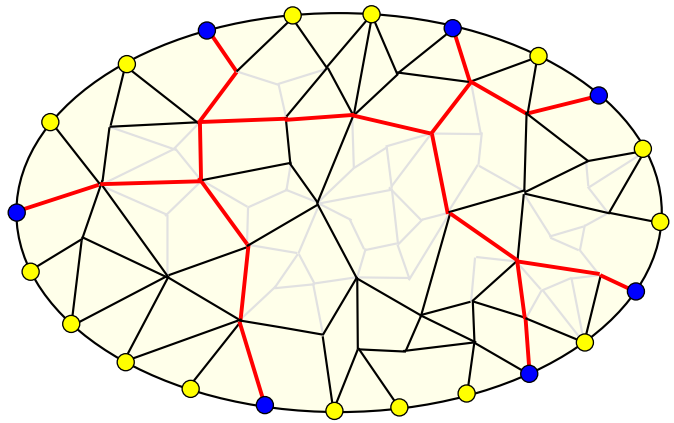

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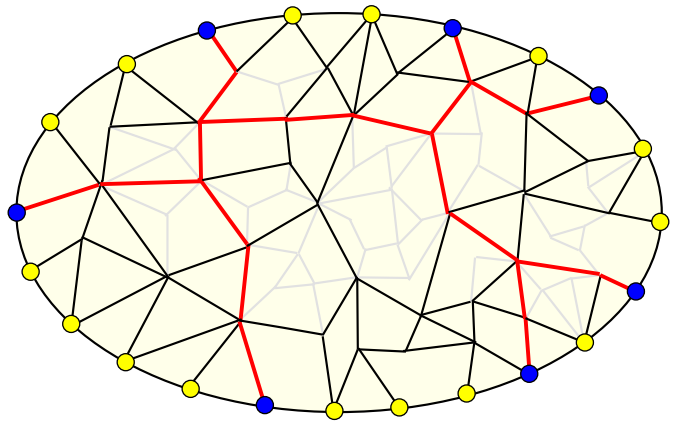

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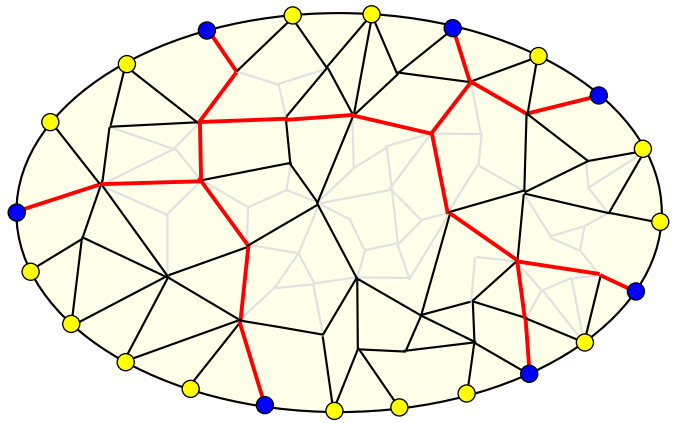

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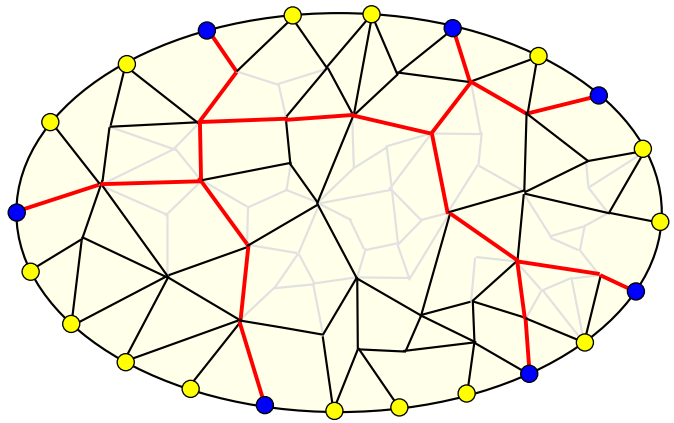

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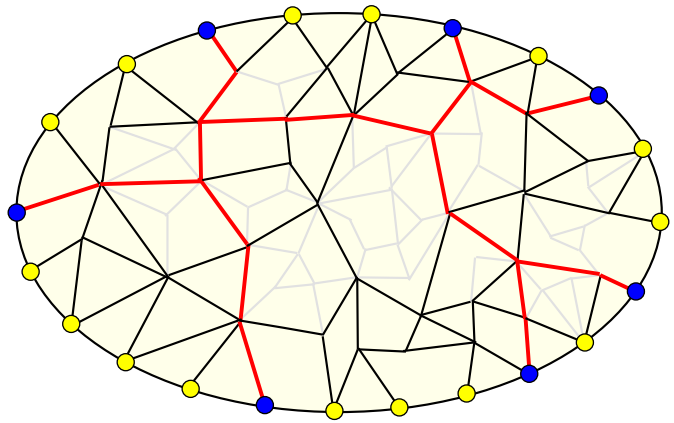

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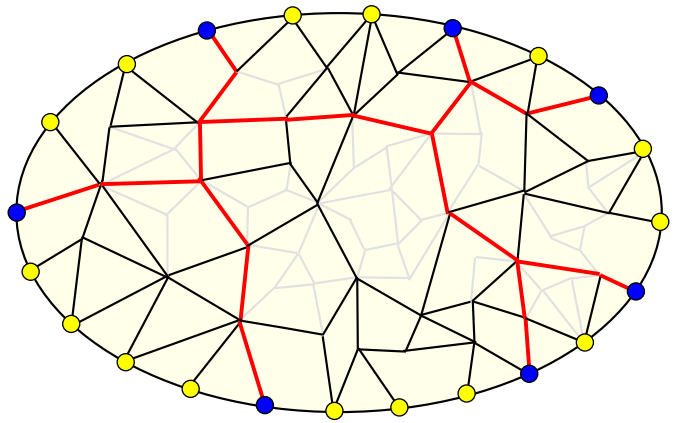

Commercial break

Some of this, and many more in the new book

Parameterized algorithms

by Cygan, Fomin, Kowalik, Lokshtanov,
Marx, Pilipczuk, Pilipczuk, and Saurabh.

Published by Springer, 2015.